CHAPTER 4
WORK AND ENERGY.

4.1 WORK DONE BY A CONSTANT FORCE:

WORK. The work done on a body by a constant force is defined as:

Definition: The product of the magnitudes of the displacement and the component of the force in the direction of the displacement.

Mathematically:

\[ W = F \cdot d = F \cdot d \cos \theta = d (F \cos \theta) \]

i.e., the scalar or dot product of force and displacement is called work, where the quantity \( F \cos \theta \) is the component of the force \( \vec{F} \) in the direction of the displacement \( \vec{d} \).

Work is a scalar quantity.

Explanation:
Consider an object which is being pulled by a constant force \( \vec{F} \) at an angle \( \theta \) to the direction of motion. Work is said to be done when this force causes a displacement in the body.

The work is also equal to the product of the magnitude of the force and the component of the displacement along the direction of the force (fig. 4.2).

\[ W = F \cdot d = F (d \cos \theta) \]

\[ \text{fig. 4.2} \]

For \( \vec{F} \) and \( \vec{d} \):

\[ d \cos \theta = d \sin \theta (\text{P.T.O.}) \]
UNIT. The SI unit of work is $\text{Nm}$ or joule (J), named after James Prescott Joule (1818-1889).

Def — "Work done will be equal to $1\text{J}$, when a force of $1\text{N}$ acting on a body of mass $1\text{kg}$ displaces it through a distance of $1\text{m}$ along the direction of force."

Dimension: $\text{Work} = N \times m = \text{kg} \times \text{m} \times \text{sec}^{-2} \times \text{m} = \text{kg} \times \text{m}^2 \times \text{sec}^{-2}$

Dim. of work $=[W]=[\text{ML}^2\text{T}^{-2}]$.

EXPLANATION OF WORK DONE BY GRAPH.

When a constant force acts through a distance $d$, the event can be plotted on a simple graph by taking distance along $x$-axis and the force along $y$-axis. As the force does not vary, the graph will be a horizontal straight line. If the constant force $F$ (newton) and the displacement $d$ (metre) are in the same direction then the work done is $Fd$ (joule).

Hence the area under a force-displacement curve can be taken to represent the work done by the force.

In case the force $F$ is not in the direction of displacement, the graph is plotted between $F\cos\theta$ and $d$.

SPECIAL CASES:

1. **Positive Work.** If the angle between force and displacement is less than $90^\circ$, i.e., $\theta < 90^\circ$, then the work is said to be positive.

   $$W = F \cdot d \cdot \cos\theta$$

   $$W = Fd \text{ (max value) for } \cos\theta = 1$$

   i.e.; Work is positive when force and displacement (P.T.O.)
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are in the same direction or the force has some component in the direction of displacement.

2. **Negative work.** If the angle between force and displacement is greater than $90^\circ$, i.e., $\theta > 90^\circ$, then the work is said to be negative.

**E.g.:**

(i) If a bucket of water is lifted against gravity very slowly, the angle between gravity and displacement is $180^\circ$. Hence,

$$W = F \cdot d = Fd \cos 180^\circ = -Fd.$$

(ii) The body moves against the force of friction on a horizontal plane.

3. **Zero work.** If the angle between force and displacement is $90^\circ$, then no work is done.

$$W = F \cdot d = Fd \cos 90^\circ = 0 \quad \Rightarrow \cos 90^\circ = 0$$

**E.g.:** The supporting force exerts to balance the weight of the bucket is vertical while the displacement is horizontal. So the supporting force does not work on the bucket.

\[ \begin{align*}
\textbf{\#4.2 WORK DONE BY A VARIABLE FORCE:} \\
\text{In many cases the force does not remain constant during the process of doing work.} \\
\text{For example:} 1. \text{As rocket moves away from the Earth, work is done against the force of gravity which varies as the inverse square of the distance from Earth's centre.} \\
2. \text{The force exerted by a spring increases with the amount of stretch.}
\end{align*} \]
EXPLANATION.

Consider a particle in a plane moving from point \(a\) to \(b\). The path has been divided into \(n\) short intervals of displacements \(\Delta d_1, \Delta d_2, \ldots, \Delta d_n\) and \(F_1, F_2, \ldots, F_n\) are the forces acting during these intervals. During each small interval, the force is supposed to be approximately constant. So, the work done for the first interval can be written as:

\[ \Delta W_1 = F_1 \cdot \Delta d_1 = F_1 \cos \theta_1 \Delta d_1 \]

Similarly, the work done in the second interval is:

\[ \Delta W_2 = F_2 \cdot \Delta d_2 = F_2 \cos \theta_2 \Delta d_2 \]

\[ \vdots \]

\[ \Delta W_n = F_n \cdot \Delta d_n = F_n \cos \theta_n \Delta d_n \]

The total work done in moving the object can be calculated by adding all these terms. Therefore,

\[ W_{\text{total}} = \Delta W_1 + \Delta W_2 + \ldots + \Delta W_n \]

\[ = F_1 \cos \theta_1 \Delta d_1 + F_2 \cos \theta_2 \Delta d_2 + \ldots + F_n \cos \theta_n \Delta d_n \]

or

\[ W_{\text{total}} = \sum_{i=1}^{n} F_i \cos \theta_i \Delta d_i \]

(1)

GRAPHICAL EXPLANATION

We can examine this graphically by plotting \(F \cos \theta\) versus \(\Delta d\) as shown in fig (b). The displacement \(\Delta d\) has been subdivided into the same \(n\) intervals. The value of \(F \cos \theta\) at the beginning of each interval is indicated by the horizontal lines.
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Now the i-th shaded rectangle has an area $F_i \cos \theta_i \Delta d_i$ which is the work done during the i-th interval. So eq. (1) gives work done which is approx equal to the sum of areas of all the rectangles as shown in fig. (b).

To get a more accurate result, we divide the total displacement into a very large number of equal intervals such that each $\Delta d$ approaches zero. In this case the work done is:

$$W_{\text{Total}} = \lim_{\Delta d \to 0} \sum_{i=1}^{n} F_i \cos \theta_i \Delta d_i \quad (2)$$

Thus, the work done by a variable force in moving a particle between two points is equal to the area under the $F \cos \theta$ verses $d^2$ curve between the two points $a^2$ and $b^2$ as shown in fig. (c).

**EXAMPLE 4.1**

A force $F$ acting on an object varies with distance $x$ as shown in fig. Calculate the work done by the force as the object moves from $x = 0$ to $x = 5$ m.

**Sol:**

Work done represented by the area of rectangle

$$= 4 \times 5 N = 20 Nm = 20 J \quad (1)$$

Work done represented by the area of triangle

$$= \frac{1}{2} \times 2 \times 5 N = 5 J \quad (2)$$

Therefore, the total work done is $20 J + 5 J = 25 J$. 

**Area of $\Delta = \frac{1}{2} \times \text{base} \times \text{height}.**
#4.3 WORK DONE IN GRAVITATIONAL FIELD.

**GRAVITATIONAL FIELD.**

Def—"The space around the Earth in which the gravitational force acts on a body is called the gravitational field."

When an object is moved in the gravitational field, the work is done by the gravitational force. If displacement is in the direction of gravitational force, the work is positive; if against, the work is negative.

**WORK DONE IN GRAVITATIONAL FIELD.**

Let us consider an object of mass \( m \) being displaced with constant velocity from point \( A \) to \( B \) along various paths in the presence of a gravitational force (fig a). In this case the gravitational force is equal to the weight \( mg \) of the object.

1—Work done along the path ADB.

The work done by the gravitational force along the path ADB can be split into two parts:

(a) The work done along \( AD \) is zero, because the weight \( mg \) is perpendicular to this path.

(b) The work done along \( DB \) is \(-mg\ h\) because the direction of \( mg \) is opposite to that of the displacement, i.e., \( \theta = 180^\circ \).

Hence, the work done in displacing a body from \( A \) to \( B \) through path 1 is:

\[
W_{AD, DB} = 0 + (-mg\ h) = -mg\ h \quad (1)
\]
2 — Work done along the path ACB.

If we consider the path ACB, it can also be split up into two parts:

(a) The work done along the path AC is also: \(-m \cdot g \cdot h\)

(b) The work done along the path CB is zero because the angle between weight and path is 90°.

Thus, the work done in displacing the object from A' to B' by path 2 is:

\[ W_{ACB} = -m \cdot g \cdot h + 0 = -m \cdot g \cdot h \]  \((2)\)

3 — Work done along the curved path AB.

Let us now consider path 3, i.e., the curved one. Imagine the curved path to be broken down into a series of horizontal and vertical steps as shown in fig. (b). There is no work done along the horizontal steps because \(mg\) is perpendicular to the displacement for these steps. Work is done by the force of gravity only along the vertical displacements. Thus

\[ W_{AB} = -mg \sum (\Delta y_1 + \Delta y_2 + \ldots + \Delta y_n) \]

As \(\Delta y_1 + \Delta y_2 + \ldots + \Delta y_n = h\)

Hence, \( W_{AB} = -m \cdot g \cdot h \) \((3)\)

**Conclusion.**

"Work done in the Earth's gravitational field is independent of the path followed."
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- **CONSERVATIVE FIELD:**
  Def—“The field in which the work done is independent of the path followed or work done in a closed path is zero is called a conservative field.”

  **For example:**
  1. Gravitational field
  2. Electric field
  3. Magnetic field etc.

- **CLOSED PATH:**
  Def—“A path in which a body after passing through several points reaches the starting point is called a closed path or loop.”

- **Conservative Forces:**
  1. Gravitational force
  2. Elastic spring force
  3. Electric force

- **NON CONSERVATIVE FIELD:**
  Def—“The field in which the work done in moving a body between two points depends upon the path followed between the two paths.”

  The *frictional force* is a non-conservative force, because if an object is moved over a rough surface between two points along different paths, the work done against the frictional force certainly depends on the path followed.

- **Non Conservative Forces:**
  1. Air resistance
  2. Tension in a string
  3. Normal force
  4. Propulsion force of a rocket
  5. Propulsion force of a motor.
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### 4.4 POWER

**Definition:** "Rate of doing work is called power." 

**Mathematically:** If work $\Delta W$ is done in a time interval $\Delta t$, then the average power during the interval $\Delta t$ is:

$$ P_{av} = \frac{\Delta W}{\Delta t} \quad (1) $$

**Instantaneous Power:**

If the rate of doing work is not uniform and work is expressed as a function of time, then the instantaneous power at any instant is defined as:

**Definition:** "Ratio of the work done to the time interval when both are extremely small is called instantaneous power."

$$ P_{inst} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} \quad (2) $$

When the work is done at uniform rate, the average power and instantaneous power are equal.

**Units:**

1. The SI unit of power is **watt (W)** after James Watt.

   **Definition:** "If one joule of work is done in one second then power will be one watt."

   $$ 1 \text{ Watt} = 1 \text{ J} \text{ s}^{-1} \quad (3) $$

   - 1 Kilowatt $= 1 \text{ kW} = 1000 \text{ W}$ (Bigger unit of power)
   - 1 Megawatt $= 1 \text{ MW} = 10^6 \text{ W}$
   - 1 Gigawatt $= 1 \text{ GW} = 10^9 \text{ W}$

2. In British Engineering system, the unit of power is **horsepower (hp)**.

   **Definition:** "When work is done at the rate of 550 ft.lbf in one second is called horsepower."

   $$ 1 \text{ hp} = 550 \text{ ft.lbf} = 746 \text{ watts} \quad (4) $$

**Units of Work in Terms of Power.**

In electrical measurements, the unit of work is expressed as watt-second. However, a commercial unit of electrical energy is a **kilowatt-hour (kWh)**. It is
also called as Board of trade unit (BOTU).

Def. "One kilowatt hour is the work done in one hour by an agency whose power is one kilowatt."

\[ 1 \text{ kWh} = 1000 \text{ W} \times 3600 \text{ S} = 3.6 \times 10^6 \text{ WS} = 3.6 \times 10^5 \text{ J} = 3.6 \text{ MJ} \]

**DIMENSIONS OF POWER.**

\[
\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{1 \text{ J}}{1 \text{ S}} = \frac{\text{M} \text{L}^2 \text{T}^{-2}}{\text{T}}
\]

\[
\text{Dim. of Power} = [P] = [\text{Ml}^2 \text{T}^{-3}]
\]

**RELATION BETWEEN FORCE, POWER AND VELOCITY.**

It is sometimes convenient to express power in terms of constant force \( \vec{F} \) acting on an object moving at constant velocity \( \vec{v} \). For example, when the propeller of a motor boat causes the water to exert a constant force \( \vec{F} \) on the boat, it moves with a constant velocity \( \vec{v} \).

As

\[
P = \text{Limit}_{\Delta t \to 0} \frac{\Delta W}{\Delta t} \quad \text{(7)}
\]

we know \( \Delta W = \vec{F} \cdot \Delta \vec{d} \quad \text{(8)} \)

Putting values, we have

\[
P = \text{Limit}_{\Delta t \to 0} \frac{\vec{F} \cdot \Delta \vec{d}}{\Delta t}
\]

Hence,

\[
P_{\text{in}} = \text{Limit}_{\Delta t \to 0} \frac{\vec{F} \cdot \vec{v}}{\Delta t}
\]

or

\[
P = \vec{F} \cdot \vec{v}
\]

i.e., Power is the dot product of force and velocity.

**EXAMPLE 4.2**

A 70 kg man runs up a long flight of stairs in 45 s. The vertical height of the stairs is 4.5 m. Calculate his power output in watts.

**Sol.**

\[
\text{Power} = \frac{mgh}{t} = \frac{70 \text{ kg} \times 9.8 \text{ m/s}^2 \times 4.5 \text{ m}}{45 \text{ s}}
\]

\[
P = 7.7 \times 10^2 \text{ W} = 7.7 \times 10^2 \text{ W}
\]
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# 4.5 ENERGY:

Def—“Capacity to do work is called energy.”

MECHANICAL ENERGY.

Def—“Energy possessed by a body due to its state of rest or motion is called mechanical energy.”

TYPES. There are two types of mechanical energy.

1. Kinetic energy (K.E)
2. Potential energy (P.E)

1. K.E.

Def—“Energy possessed by a body due to its motion is called K.E.”

Formula: \[ K.E = \frac{1}{2}mv^2 \]  
or \[ K.E = \frac{1}{2}m(v^2 - v_0^2) \]

2. P.E.

Def—“Energy possessed by a body due to its position in a force field e.g gravitational field or because of its constrained state is called P.E.”

Formula: The P.E of a body due to gravitational field near the surface of the Earth at a height \( h \) can be expressed as:

\[ P.E = mgh \]

This is called gravitational P.E, which is always determined relative to some arbitrary position which is assigned the value of zero P.E. The Earth’s surface or a point at infinity from the Earth can be chosen as zero reference level of gravitational P.E. In present case, this reference level is the surface of the Earth as position of zero P.E.

UNITS. The SI unit of energy is joule (J).

DIMENSION. [Dim. of energy] = [ML^2T^{-2}]
ELASTIC P.E.

Def. “The energy possessed by the spring due to its compressed or stretched state is called elastic P.E.”

· WORK ENERGY PRINCIPLE:

Statement “Work done on the body is equal to the change in its K.E.”

EXPLANATION.

Whenever work is done on a body, it increases its energy. Suppose a body of mass \(m\) is moving with velocity \(v_i\). A force \(F\) acting through a distance \(d\) increases the velocity to \(v_f\), then from 3rd equation of motion:

\[
2ad = v_f^2 - v_i^2
\]

or

\[
d = \frac{v_f^2 - v_i^2}{2a}
\]  \hspace{1cm} (1)

From second law of \(2\)nd motion:

\[F = ma\] \hspace{1cm} (2)

Multiplying equ. \(1\) and \(2\), we get:

\[Fd = \frac{1}{2}m(v_f^2 - v_i^2)\]

or

\[Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\]

\[Fd = (K.E)\_f - (K.E)\_i\] \hspace{1cm} (3)

As \(Fd\) = Work done on the body,

\[W = (K.E)\_f - (K.E)\_i\]

\[W = \text{change in K.E of the body.}\]

This is called \textbf{work-energy principle}.

\textbf{N.B.} If work is positive, the final K.E is greater than the initial K.E and K.E increases. If the work is negative, the K.E decreases. When work is zero, the K.E remains constant.

If a body is raised up from Earth’s surface, the work done increases the gravitational P.E. \textbf{Similarly}, if a spring is compressed, the work done on it equals the increase in its elastic P.E.
# ABSOLUTE POTENTIAL ENERGY:

Def. "The work done in moving a body from Earth's surface to a point far away from the Earth where the value of $g$ is negligible is called Absolute P.E."

**DETERMINATION OF A.P.E.**

The relation for the calculation of the work done by the gravitational force or $P.E = mgh$, is true only near the surface of the Earth where the gravitational force is constant. But if the body is displaced through a large distance in space from, let, point 1 to $N$ (fig.) in the gravitational field, then the gravitational force will not remain constant, since it varies inversely to the square of the distance. So, we divide the distance between 1 and $N$ into small steps each of length $\Delta r$ such that the value of the force remains constant for each small step. Hence, the total work done can be calculated by adding the work done during all these steps. If $r_1$ and $r_2$ are the distances of points 1 and 2 respectively, from the centre $O$ of the Earth, then the work done during the first step i.e., displacing a body from point 1 to point 2 can be calculated as:

The distance between the centre of the step and the centre of the Earth will be:

$$r = \frac{r_1 + r_2}{2}$$  \(1\)

If $r_2 - r_1 = \Delta r$  \(2\)

then $r_2 = \Delta r + r_1$  \(3\)
Putting the value of \( r_0 \) in eq (1), we have
\[
r = \frac{r_1 + r_2 + \Delta r}{2} = \frac{2r_1 + \Delta r}{2}
\]
\[
r = r_1 + \frac{\Delta r}{2} \quad (4)
\]
Squaring both sides, we have;
\[
r^2 = r_1^2 + \frac{\Delta r^2}{4} + r_1 \Delta r \quad (5)
\]
As \((\Delta r)^2<<r_1^2\), so this term can be neglected as \(r_1\)
Hence
\[
r^2 = r_1^2 + r_1 \Delta r \quad (6)
\]
Putting the value of \( \Delta r \), we have
\[
r^2 = r_1^2 + r_1 (r_2 - r_1)
\]
\[
= r_1^2 + r_1 r_2 - r_1^2
\]
\[
r^2 = r_1 r_2 \quad (7)
\]
The gravitational force \( F \) at the centre of the step is,
\[
F = \frac{G M m}{r^2} \quad (8)
\]
where \( M \) = mass of Earth, \( G \) = gravitational constant, \( m \) = mass of an object.
becomes;
\[
F = \frac{G M m}{r_1 r_2} \quad (9)
\]
As this force is assumed to be constant during the interval \( \Delta r \), so the work done is
\[
W_{1 \rightarrow 2} = F \cdot \Delta r = F \Delta r \cos 180^\circ = -\frac{G M m \cdot \Delta r}{r_1 r_2} \quad (10)
\]
The negative sign indicates that the work has to be done on the body from point 1 to 2 because displacement is opposite to gravitational force.
Putting the value of \( \Delta r \), we get
\[
W_{1 \rightarrow 2} = -\frac{G M m \cdot (r_2 - r_1)}{r_1 r_2}
\]
\[
= -\frac{G M m}{r_1 r_2} \left[ 1 - \frac{1}{r_2} \right]
\]
Similarly;
\[
W_{2 \rightarrow 3} = -\frac{G M m}{r_2 r_3} \left[ 1 - \frac{1}{r_3} \right]
\]
\[
\vdots
\]
\[
W_{(n-1) \rightarrow n} = -\frac{G M m}{r_{n-1} r_n} \left[ 1 - \frac{1}{r_n} \right]
\]
Hence, the total work done in displacing a body from point 1 to \( N \) is calculated by adding up the work done during all these steps.

\[
W_{\text{total}} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + \cdots + W_{(N-1) \rightarrow N}
\]

\[
= -GMm \left[ \left( \frac{1}{r_1} - \frac{1}{r_2} \right) + \left( \frac{1}{r_2} - \frac{1}{r_3} \right) + \cdots + \left( \frac{1}{r_{N-1}} - \frac{1}{r_N} \right) \right]
\]

On simplification, we get

\[
W_{\text{total}} = -GMm \left[ \frac{1}{r_N} \right] \tag{10}
\]

If the point \( N \) is situated at an infinite distance from the Earth, so

\[
r_N = \infty \quad \text{then} \quad \frac{1}{r_N} = \frac{1}{\infty} = 0
\]

Hence,

\[
W_{\text{total}} = -GMm
\]

Therefore, the general expression for the gravitational P.E of a body situated at distance \( r \) from the Earth is

\[
U = -GMm \tag{11}
\]

This is known as the absolute value of gravitational P.E of a body at a distance \( r \) from the centre of Earth.

Now, when \( r \) increases, the gravitational force does negative work and \( U \) increases (i.e., becomes less negative).

When \( r \) decreases, the body falls towards the Earth, the work is positive and P.E decreases (i.e., becomes more negative). \( U \) is zero when \( (r = \infty) \), the mass \( m' \) is infinitely away from the Earth, so it is quite a different choice for making \( U = 0 \) at some arbitrary position. Therefore, the choice of zero point is arbitrary and the only difference of P.E from one point to another is significant, so negative values of \( U \) should not be too alarming.
Now the absolute P.E. on the surface of the Earth is found by putting \( r = R \) (Radius of the Earth)

\[
A.P.E = U_e = -\frac{G M m}{r}
\]

The negative sign shows that the Earth's gravitational field for mass \( m \) is attractive. The above eq. gives the work or the energy required to take the body out of the Earth's gravitational field, where its P.E. w.r.t. Earth is zero.

**# ESCAPE VELOCITY:**

When an object is projected upward, it comes back to ground after rising to a certain height. This is due to the force of gravity acting downward. With increased initial velocity, the object rises to the greater height before coming back. If we go on increasing the initial velocity of the object, a stage comes when it will not return to the ground. It will escape out of the influence of gravity of the Earth.

Def. — “The initial velocity of an object with which it goes out of the Earth's gravitational field is known as escape velocity.”

**EXPRESSION FOR ESCAPE VELOCITY.**

The escape velocity corresponds to the initial K.E. gained by the body, which carries it to an infinite distance from the surface of Earth is

\[
\text{Initial K.E.} = \frac{1}{2} m v_e^2
\]

The work done in lifting a body from Earth's surface to an infinite distance is equal to the absolute P.E. of the body at Earth's surface is

\[
A.P.E. = \frac{G M m}{R}
\]
The body will escape out of the gravitational field if the initial K.E of the body is equal to the absolute P.E. Then

\[ \frac{1}{2} m v_{esc}^2 = \frac{GMm}{r} \]

or

\[ v_{esc} = \sqrt{\frac{2GM}{r}} \quad (3) \]

As

\[ \epsilon = \frac{GM}{r^2} \Rightarrow \epsilon R = \frac{GM}{r} \]

Hence,

\[ v_{esc} = \sqrt{2\epsilon R} \quad (4) \]

Putting value, we have

\[ v_{esc} = \sqrt{2 \times 9.8 \text{ m/s}^2 \times 3.4 \times 10^6} \]

\[ = 11 \times 10^3 \text{ m/s} \]

\[ v_{esc} = 11 \text{ Km/s} \]

## 4.6 INTERCONVERSION OF P.E AND K.E

Consider a body of mass ‘m’ at rest at a height ‘h’ above the surface of the Earth.

**AT POINT ‘A’**.

\[ (P.E)_{A} = mgh \quad (1) \]

and \( (K.E)_{A} = 0 \quad (2) \)

So \( (T.E)_{A} = mgh \quad (3) \)

When the body is dropped from a certain height, its K.E and P.E are interchange.

**AT POINT ‘B’**.

The P.E and K.E at the position ‘B’ when the body has fallen a distance ‘x’, ignoring air friction is:

\[ (P.E)_{B} = m \epsilon (h - x) \quad (4) \]

and \( (K.E)_{B} = \frac{1}{2} m v_B^2 \quad (5) \)

Velocity \( v_B \), at ‘B’ can be calculated from the relation,
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\[ v_f^2 = v_i^2 + 2gs \]
\[ v_f = v_i \Rightarrow v_i = 0 \Rightarrow S = x \Rightarrow \text{we have;} \]
\[ v_B^2 = 0 + 2gs = 2gs \quad \text{(3)} \]
\[ \therefore (K.E)_B = \frac{1}{2}m(2gs) = mgs \quad \text{(7)} \]
\[ (T.E)_B = (P.E)_B + (K.E)_B \]
\[ (T.E)_B = mgh - x + mgs \]
\[ (T.E)_B = mgh \quad \text{(8)} \]

\textbf{AT POINT 'C'}

At position 'C', just before the body strikes the Earth:
\[ (P.E)_C = 0 \quad \text{(9)} \]
\[ (K.E)_C = \frac{1}{2}m v_c^2 \quad \text{(10)} \]
where \( v_c \) can be found out by the following rel:
\[ v_f^2 = v_i^2 + 2gs \]
\[ v_f = v_c \Rightarrow v_i = 0 \Rightarrow S = h \]
\[ v_c^2 = 2gh \quad \text{(11)} \]
\[ \therefore (K.E)_C = \frac{1}{2}m \times 2gh = mgh \quad \text{(12)} \]
\[ (T.E)_C = (P.E)_C + (K.E)_C \]
\[ = 0 + mgh \]
\[ (T.E)_C = mgh \quad \text{(13)} \]

Thus at point 'C', K.E is equal to the original P.E of the body. Actually when a body falls, its velocity increases i.e., the body is being accelerated under the action of gravity. The increase in velocity results in the increase in its K.E. On the other hand, as the body falls, its height decreases and hence, its P.E also decreases. So,

\[ \text{Loss in P.E = Gain in K.E} \]
\[ \text{or; } mgh (h_1 - h_2) = \frac{1}{2}m (v_2^2 - v_1^2) \quad \text{(14)} \]

Where \( v_1 \) and \( v_2 \) are velocities of the body at the heights \( h_1 \) and \( h_2 \) respectively, when no friction is involved.
WHEN FRICTIONAL FORCE IS PRESENT.

If we assume that a frictional force $f$ is present during the downward motion, then a part of P.E is used in doing work against friction equal to $fh$. The remaining $PE = mgh - fh$ is converted in K.E.

Hence,

$$mgh - fh = \frac{1}{2}mv^2$$

or

$$mgh = \frac{1}{2}mv^2 + fh$$

Thus,

Loss in P.E = Gain in K.E + Work done against friction.

N.B After the body hitting the ground, the P.E and K.E are converted into sound and heat energies. This is called dissipation of energy.

# 4.7 LAW OF CONSERVATION OF ENERGY:

Statement — “Energy cannot be destroyed. It can be transformed from one kind into another, but the total amount of energy remains constant.”

EXPLANATION.

The kinetic and potential energies are both different forms of the same basic quantity i.e., mechanical energy. The total mechanical energy is the sum of its K.E and P.E. It is an experimental fact that one form of energy can be converted into another form. E.g., an electrical and chemical energy are more easily transferred into heat. Also the P.E of the falling object changes to K.E but on striking the ground, the K.E changes into heat and sound. If it seems in an energy transfer that some energy has disappeared, the lost energy is often converted into heat. Ultimately, all energy transfers result in heating of environment.
EXAMPLE 4.3: A brick of mass 2 kg is dropped from a rest position 5 m above the ground. What is its velocity at a height of 3 m above the ground?

**DATA**  \( m = 2 \text{ kg} \), initial height \( h_1 = 5 \text{ m} \),

final height \( h_2 = 3 \text{ m} \), \( g = 9.8 \text{ m/s}^2 \), \( V_1 = 0 \)

\( V_2 \) = velocity of brick at height \( h_2 = ? \)

**Sol.** As \( \text{Loss in P.E} = \text{Gain in K.E} \)

\[ F_\text{g} (h_1 - h_2) = \frac{1}{2} m (V_2^2 - V_1^2) \]

\[ 9.8 \text{ m/s}^2 (5 \text{ m} - 3 \text{ m}) = \frac{1}{2} (V_2^2 - 0) \]

\[ V_2^2 = 2 \times 9.8 \times 2 \]

\[ V_2 = \sqrt{2 \times 9.8 \times 2} = \sqrt{39.2} \text{ m/s} \]

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# 4.8 NON CONVENTIONAL ENERGY SOURCES:

Energy is being consumed at an enormous rate in the world today. The present annual consumption is about \( 6 \times 10^{13} \text{kWh} \). Scientists are always exploring new sources of energy to fulfill the future needs. Some of these are as:

1. **ENERGY FROM TIDES.**

   Energy can be obtained from tides. Gravitational force of the moon gives rise to tides in the sea. The tides raise the water in the sea roughly twice a day. If the water at the high tide is trapped in a basin by constructing a dam, then it is possible to use this as a source of energy. The dam is
filled at high tide and water is released in a controlled way at low tide to drive the turbines. At the next high tide the dam is filled again and the rushing water also drives turbines and generates electricity as shown.

Tidal power stations have been installed in France, Alaska, Argentina etc.

2—ENERGY FROM WAVES.

The tidal movement and the winds blowing across the surface of the ocean produce strong water waves. Their energy can be utilized to generate electricity. A method of harnessing wave energy is to use large floats which move up and down with the waves. One such device invented by Professor Salter is known as Salter’s duck. It consists of two parts:

(i) Duck float
(ii) Balance float

The wave energy makes the duck float move relative to the balance float. The relative motion of the duck float is then used to run electricity generators.
3. SOLAR ENERGY

The Earth receives huge amount of energy directly from the sun each day. Energy from Sun is called solar energy. Every year the earth absorbs about $4 \times 10^7$ KWh of solar energy. If we could utilize only 0.1% of the incident solar energy, it would be more than sufficient for the entire world’s energy requirements. Solar energy at normal incidence outside the Earth’s atmosphere is about 1.4 KWM$^{-2}$ which is referred as solar constant. While passing through the atmosphere, the total energy is reduced due to reflection, scattering and absorption by dust particles, water vapours and other gases. On a clear day at noon, the intensity of the solar energy reaching the Earth’s surface is about 1 KWM$^{-2}$. This energy can be used directly to heat water using large solar reflectors and thermal absorbers or be converted to electricity.

In one method, the flat plate collectors are used for heating water. A typical collector is shown in Fig. It has a blanckened surface which absorbs energy directly from solar radiation. Cold water passes over the surface and is heated up to about 70°C. Much higher temp. can be achieved by concentrating solar radiation onto a small surface area by using huge reflectors (mirrors) or lenses to produce steam for running a turbine.
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The other method is the direct conversion of sunlight into electricity through the use of semiconductor devices called solar cells also known as photovoltaic cells.

Def: "A solar cell is a device which is used to convert solar energy into electric energy."

Solar cells are thin wafers made from silicon. Electrons in the silicon gain energy from sunlight to create a voltage. The voltage produced by a single photovoltaic cell is very low. In order to get sufficient high voltage for practical use, a large no. of such cells are connected in series forming a solar cell panel.

For cloudy days or nights, electric energy can be stored during the sun light in nickel cadmium batteries by connecting them to solar panels. These batteries can then provide power to electrical appliances at nights or on cloudy days.

Solar cells, although, are expensive but last a long time and have low running cost.

USES OF SOLAR CELLS

(i) Solar cells are used to power satellites having large solar panels which are kept facing the Sun.

(ii) Solar cells are used for remote ground based weather stations and rain forest communication systems.

(iii) Solar calculators are also in use, etc.

4 ENERGY FROM BIOMASS:

Biomass is a potential source of renewable energy. This includes all the organic materials such as crop residue, natural vegetation, trees, animal dung and sewage. Biomass energy or
Bio conversion refers to the use of this material as fuel or its conversion into fuels. There are many methods used for the conversion of biomass into fuels. But the most common are:

(i) Direct combustion
(ii) Fermentation.

Direct combustion method is usually applied to get energy from waste products commonly known as solid waste.

- Biofuel such as ethanol (alcohol) is a replacement of gasoline. It is obtained by fermentation of biomass using enzymes and by decomposition through bacterial action in the absence of air (oxygen).

- The rotting of biomass in a closed tank called a digester produces biogas which can be piped out to use for cooking and heating (Fig).

- The waste material of the process is a good organic fertilizer. Thus, production of biogas provides us energy source and also solves the problem of organic waste disposal.

Energy from Waste Products:

Waste products like wood waste, crop residue, and particularly municipal solid waste can be used to get energy by direct conversion. It is probably the most commonly used conversion process in which waste material is burnt in a confined container. Heat produced in this way is directly utilized in the boiler to produce steam that can run turbine generator.
6. GEOTHERMAL ENERGY:

This is the heat energy extracted from inside the Earth in the form of hot water or steam. Heat within the Earth is generated by the following processes.

(i) Radioactive Decay

The energy, heating the rocks, is constantly being released by the decay of radioactive elements.

(ii) Residual heat of the Earth

At some places hot igneous rocks, usually within 10 km of the Earth's surface, are in a molten and partly molten state. They conduct heat energy from the Earth's interior which is still very hot. The temp. of these rocks is about 200°C or more.

(iii) Compression of material

The compression of material deep inside the Earth also causes generation of heat energy.

In some places water beneath the ground is in contact with hot rocks and is raised to high temp. and pressure. It comes to the surface as hot springs, geysers, or steam vents. The steam can be directed to turn turbines of electric generators. Where water is not present and hot rocks are not very deep, the water is pumped down through them which returns as steam (fig). The steam then can be used to drive turbines or for direct heating.

An interesting phenomenon of geothermal energy is a geysers. It is a hot spring that discharges steam and hot water intermittently.
releasing an explosive column into the air. Most geysers erupt at irregular intervals. They usually occur in volcanic regions. Extraction of geothermal heat energy often occurs closer to geyser sights. This extraction seriously disturbs geyser system by reducing heat flow and aquifer pressure. Aquifer is a layer of rock holding water that allows water to percolate through it with pressure.
SHORT QUESTIONS

Q 4.1 A person holds a bag of groceries while standing still, talking to a friend. A car is stationary with its engine running. From the standpoint of work, how are these two situations similar?

Ans: In both cases, the displacement is zero and hence, no work is done, so these situations are similar.

Q 4.2 Calculate the work done in kilo joules in lifting a mass of 10 kg (at a steady velocity) through a vertical height of 10 m.

Ans: In this case, work done is stored in the body as its P.E. so

\[ \text{Work} = P.E = mgh = 10 \, \text{kg} \times 9.8 \, \text{m/s}^2 \times 10 \, \text{m} = 980 \, \text{J} = 0.98 \, \text{kJ} \]

Hence, approx. 1 kJ work is done.

Q 4.3 A force \( F \) acts through a distance \( L \). The force is then increased to \( 3F \), and then acts through a further distance of \( 2L \). Draw the work diagram to scale.

Ans: A force \( F \) acts through a distance \( L \), then work done = \( FL \) \[ 1 \)

If the force is increased to \( 3F \) through a distance \( 2L \), then work done is \( 3F \times 2L = 6FL \) \[ 2 \)

The area under graph is equal to the total work done. So

\[ \text{Total work done} = FL + 6FL = 7FL \, \text{(units)} \]
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Q. 4.4 In which case is more work done? When a 50 kg bag of books is lifted through 50 cm, or when a 50 kg crate is pushed through 2 m across the floor with a force of 50 N?

\textbf{Ans:} (a) 
\[ m = 50 \text{ kg}, \quad h = 50 \text{ cm} = 0.5 \text{ m} \]
In this case work done is stored as its P.E.,
\[ W_1 = P.E. = mgh \]
\[ = 50 \times 9.8 \times 0.5 \]
\[ = 245 \text{ J} \] \(\text{(1)}\)

(b) 
\[ m = 50 \text{ kg}, \quad d = 2 \text{ m}, \quad F = 50 \text{ N} \]
\[ W_2 = F \cdot d \]
\[ = 50 \text{ N} \times 2 \text{ m} = 100 \text{ J} \] \(\text{(2)}\) \quad \therefore 1 \text{ N} \cdot \text{m} = 1 \text{ J}

So in 1st case work done is more.

Q. 4.5 An object has 1 J of P.E. Explain what does it mean?

\textbf{Ans:} An object has P.E. when it is in a force field such as gravitational field or it is in a constrained state such as a compressed spring or extended spring.

(a) When an object is raised by doing 1 J of work,
\[ P.E. = mgh = Wh = 1 \text{ J} \left( 1 \text{ N} \times 1 \text{ m} \right) \]
i.e. when one newton force is applied on a body and lift it up through a height of 1 m, the work appears as its gravitational P.E. Then the work done is stored in the body is 1 J.

(b) When a spring is compressed or extended by doing 1 J work on it, the work is conserved in the spring as its elastic P.E.

Q. 4.6 A ball of mass \( m \) \( \text{ kg} \) is held at a height \( h_1 \) above a table. The table top is at a height \( h_2 \) above the floor. One student says that the ball has P.E. = \( mgh_1 \), but another says that it is \( mgh_1 + h_2 \). Who is correct?

\textbf{Ans.} Both statements are correct. It is matter of
relative position for specifying the P.E of the ball. The P.E relative to the tabletop is $mgh_1$ and P.E relative to the ground is $mg(h_1+h_2)$.

Q 4.7 When a rocket re-enters the atmosphere, its nose cone becomes very hot. Where does this heat energy come from?

**Ans:** When a rocket re-enters the atmosphere with high velocity, its nose becomes very hot due to air friction. The work done against the air friction is changed into heat.

Q 4.8 What sort of energy is in the following:
(a) compressed spring (b) water in a high dam (c) a moving car.

**Ans:** (a) Elastic P.E is stored in a compressed spring.
(b) The water in a high dam has gravitational P.E due to its height w.r.t the basin.
(c) A moving car has K.E due to its motion.

Q 4.9 A girl drops a cup from a certain height, which breaks into pieces. What energy changes are involved?

**Ans:** The cup has gravitational P.E at a certain height. When it is dropped, its P.E decreases and K.E increases. Just before striking the floor, the whole of P.E is changed into K.E. On striking the ground, the K.E is changed into sound, heat and work to break the cup into pieces.

Q 4.10 A boy uses a catapult to throw a stone which accidentally smashes a greenhouse window. List the possible energy changes.

**Ans:** The elastic P.E of the catapult is transferred to stone as its K.E. When the stone strikes the greenhouse window, the K.E of the stone is changed into sound, heat and work done in breaking the window glass.
NUMERICAL PROBLEMS

P. 4.1 A man pushes a lawn mower with a 40 N force directed at an angle of 20° downward from the horizontal. Find the work done by the man as he cuts a strip of grass 20 m long.

 DATA. \( F = 40 \text{ N} \)
\( \theta = 20^\circ \)
\( d = 20 \text{ m} \)

 Work done = \( W = ? \)

 Sol.: using \( W = F \cdot d \cos \theta \)
\[ W = 40 \text{ N} \cdot 20 \text{ m} \cdot \cos 20^\circ \]
\[ = 7.5 \times 10^2 \text{ J} \]

P. 4.2 A rain drop (\( m = 3.35 \times 10^{-5} \text{ kg} \)) falls vertically at a constant speed under the influence of the forces of gravity and friction. In falling through 100 m, how much work is done by (a) gravity and (b) friction.

 DATA. \( m = 3.35 \times 10^{-5} \text{ kg} \)
Height = \( h = 100 \text{ m} \)

(a) Work done due to gravity = ?

(b) Work done due to friction = ?

 Sol.: we have

 Work done due to gravity = \( W = mgh \)
\[ W = 100 \text{ m} \times 9.8 \text{ m}^2/\text{s}^2 \times 3.35 \times 10^{-5} \text{ kg} \]
\[ = 0.0328 \text{ J} \]

 Work done due to both gravity and friction is the same. Here, the frictional force is acting on the rain drop against the gravitational force. So,

 Work done by friction = \( -0.0328 \text{ J} \)
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P. 4.3 Ten bricks, each 6 cm thick and mass $1.5 \text{ kg}$, lie flat on a table. How much work is required to stack them one on the top of another?

**DATA:** Mass of each brick $= 1.5 \text{ kg}$

Thickness of brick $= h = 6 \text{ cm} = 0.06 \text{ m}$

Work required to stack them one on the top of another $= W =$

**Sol:** There is no work done for the 1st brick. We have to put bricks on it one by one. Therefore, every brick gets some P.E. w.r.t its height. So

$W = 0 + 1.5 \times 9.8 \times 0.06 = 0.57 \text{ J}$

Putting values, we have

$W = 45 \times 1.5 \times 9.8 \times 0.06 = 39.69 \text{ J} = 40 \text{ J}$

P. 4.4 An object of mass $6 \text{ kg}$ is travelling at a velocity of $5 \text{ m s}^{-1}$. What is its K.E. if its velocity is double?

**DATA:** Mass of the object $= m = 6 \text{ kg}$

Velocity of object $= V = 5 \text{ m s}^{-1}$

(a) K.E. $= ?$

(b) K.E. $= ?$ (When its velocity is double)

**Sol:** (a) We know that

$K.E = \frac{1}{2} m v^2$

$= \frac{1}{2} \times 6 \times 5^2 = 37.5 \text{ J}$

(b) Now $V = 10 \text{ m s}^{-1}$, therefore

$K.E = \frac{1}{2} m v^2$

$= \frac{1}{2} \times 6 \times 10^2 = 300 \text{ J} = 3 \times 10^2 \text{ J}$
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P.4.5 An electron strikes the screen of a cathode-ray tube with a velocity of \( 1.0 \times 10^7 \, \text{m/s} \). Calculate its K.E. The mass of an electron is \( 9.1 \times 10^{-31} \, \text{kg} \).

**DATA**
- Velocity of electron = \( V = 1.0 \times 10^7 \, \text{m/s} \)
- Mass of electron = \( m = 9.1 \times 10^{-31} \, \text{kg} \)

**K.E. = ?**

**Sol:** As we know that:

\[
\text{K.E.} = \frac{1}{2} m V^2
\]

\[
= \frac{1}{2} \times 9.1 \times 10^{-31} \, \text{kg} \times (1.0 \times 10^7 \, \text{m/s})^2
\]

\[
= 4.5 \times 10^{-17} \, \text{J}
\]

P.4.6 If \( 100 \, \text{m}^3 \) of water is pumped from a reservoir into a tank, 10 m higher than the reservoir, in 20 minutes. If density of water is \( 1000 \, \text{kg/m}^3 \) find:

(a) the increase in P.E.
(b) the power delivered by the pump.

**DATA**
- Volume of water = \( V = 100 \, \text{m}^3 \)
- Height = \( h = 10 \, \text{m} \)
- Time taken = \( t = 20 \, \text{min} = 20 \times 60 \, \text{s} = 1200 \, \text{s} \)
- Density of water = \( \rho = 1000 \, \text{kg/m}^3 \)

(a) **Increase in P.E. = ?**

(b) **Power delivered by the Pump = \( P = ? \)**

**Sol**

(a) Density (\( \rho \)) = \( \frac{\text{Mass (m)}}{\text{Volume (V)}} \)

\[
m = V \times \rho
\]

\[
= 100 \, \text{m}^3 \times 1000 \, \text{kg/m}^3
\]

\[
m = 10^5 \, \text{kg}
\]

Also, work done = \( P.E. = mgh \)

\[
= 10^5 \, \text{kg} \times 9.8 \, \text{m/s}^2 \times 10 \, \text{m}
\]

Increase in P.E. = \( 9.8 \times 10^5 \, \text{J} \)

(b) Now, power (\( P \)) = \( \frac{\text{Work (W)}}{\text{Time (t)}} \)

\[
P = \frac{9.8 \times 10^5 \, \text{J}}{1200 \, \text{s}} = 8166.6 \, \text{Watt}
\]

\[
P = 8.167 \times 10^3 \, \text{Watt} = 8.2 \, \text{KW}
\]
P.4.7 A force (thrust) of 400 N is required to overcome road friction and air resistance in propelling an automobile at 80 km h⁻¹. What power (KW) must the engine develop?

**DATA.**
- \( F = 400 \text{ N} \)
- \( \text{Velocity} = V = 80 \text{ km h}^{-1} = \frac{80 \times 10^3 \text{ m}}{3600} = 22.22 \text{ m s}^{-1} \)
- \( \text{Power} = P \text{ (in KW)} = ? \)

**Sol.**
- \( P = F \times V = FV \text{ cos} 0^\circ = FV \) (\( F \) and \( V \) are along the same direction)
- \( P = 400 \text{ N} \times 22.22 \text{ m s}^{-1} \)
- \( P = 8888 \text{ Watt} \)
- \( 8.9 \times 10^3 \text{ W} = 8.9 \text{ KW} \)

P.4.8 How large a force is required to accelerate an electron (\( m = 9.1 \times 10^{-31} \text{ kg} \)) from rest to a speed of \( 2 \times 10^7 \text{ m s}^{-1} \) through a distance of 5 cm?

**DATA.**
- Mass of electron = \( m = 9.1 \times 10^{-31} \text{ kg} \)
- Initial velocity = \( V_i = 0 \)
- Final velocity = \( V_f = 2 \times 10^7 \text{ m s}^{-1} \)
- Distance = \( d = 5 \text{ cm} = 0.05 \text{ m} \)
- Force required = \( F = ? \)

**Sol.**
- Using work-energy principle;
  \( F \times d = \frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2 \)
- Putting the values, we have;
  \( F \times 0.05 \text{ m} = \frac{1}{2} \times 9.1 \times 10^{-31} \text{ kg} \times (2 \times 10^7 \text{ m s}^{-1})^2 - 0 \)
  \( F = \frac{3.6 \times 10^{-15} \text{ N} \times 0.05 \text{ m}}{\frac{1}{2} \times (2 \times 10^7 \text{ m s}^{-1})^2} \)
  \( F = 3.6 \times 10^{-15} \text{ N} \)

P.4.9 A diver weighing 750 N drops from a board 10 m above the surface of a pool of water. Use the conservation of mechanical energy to find his speed at a point 5 m above the water surface, neglecting air friction.
**DATA.** Weight of diver = \( W = 750 \text{ N} \)
- \( h_1 = 10 \text{ m} \)
- \( h_2 = 5 \text{ m} \)

**Sol.** As we know:

- Loss of P.E = Gain in K.E
- \( m g (h_1 - h_2) = \frac{1}{2} m v^2 \)
- \( v^2 = 2 g (h_1 - h_2) \)

Putting values, we have:
- \( v = \sqrt{2 \times 9.8 \text{ m/s}^2 \times (10 \text{ m} - 5 \text{ m})} \)
- \( v = \sqrt{9.9 \text{ m/s}} \)

**P.4.10** A child starts from rest at the top of a slide of height 4 m. (a) What is his speed at the bottom if the slide is frictionless? (b) If he reaches the bottom with a speed of 6 m/s, what percentage of his total energy at the top of the slide is lost as a result of friction?

**DATA.** Height of slide = \( h = 4 \text{ m} \)
- Speed at the bottom = \( v = ? \)
- \( \% \) age of total energy lost = ? (if \( v = 6 \text{ m/s} \))

**Sol.** (a) As loss of P.E = Gain in K.E
- \( m g h = \frac{1}{2} m v^2 \)
- \( v^2 = \frac{2 g h}{2} = 2 \times 4.8 \text{ m/s}^2 \times 4 \text{ m} \)
- \( v = \sqrt{8.8 \text{ m/s}} \)

(b) Loss of energy = K.E - K.E'
- \( \frac{1}{2} m v^2 - \frac{1}{2} m v^{'2} \)
- \( = \frac{1}{2} m \times (8.8)^2 - \frac{1}{2} m \times (6)^2 \)
- \( = \frac{(38.72 \text{ m} - 18 \text{ m})}{2 \text{ J}} \)
- \( = \frac{(20.72 \text{ m})}{2 \text{ J}} \)

- \( \% \) Loss of energy = \( \frac{20.72 \text{ m} \times 100}{38.72 \text{ m}} \)
- \( = 54\% \)